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# QOMP MODEL TO ENHANCE THE QUALITY OF COLOR IMAGE

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## ABSTRACT

Sparse image models treat color image pixels as scalar, it represents color channels separately or concatenate color channels as a monochrome image. This paper proposes a novel sparse model for color image using quaternion matrix analysis. It represents color image pixels as quaternion matrix. The proposed model represents the color image as a quaternion matrix, where a quaternion-based dictionary learning algorithm is presented using the K-quaternion singular value decomposition (QSVD) (generalized K-means clustering for QSVD) method. It conducts the sparse basis selection in quaternion space, which uniformly transforms the channel images to an orthogonal color space. In this new color space, it is significant that the inherent color structures can be completely preserved during vector reconstruction. Moreover, the proposed sparse model is more efficient comparing with the current sparse models for image restoration tasks due to lower redundancy between the atoms of different color channels. The experimental results demonstrate that the proposed sparse image model avoids the hue bias issue successfully and shows its potential as a general and powerful tool in color image analysis and processing domain.

**Keywords**— *Vector sparse representation, quaternion matrix analysis, color image, dictionary learning, K-QSVD, image restoration.*

## I. INTRODUCTION

Super-resolution (SR) image reconstruction is currently a very active area of research, as it offers the promise of overcoming some of the inherent resolution limitations of low-cost imaging sensors (e.g. cell phone or surveillance cameras) allowing better utilization of the growing capability of high-resolution displays (e.g. high-definition LCDs). Such resolution-enhancing technology may also prove to be essential in medical imaging and satellite imaging where diagnosis or analysis from low-quality images can be extremely difficult. Conventional approaches to generating a super-resolution image normally require as input multiple low-resolution images of the same scene, which are aligned with sub-pixel accuracy. The SR task is cast as the inverse problem of recovering the original high-resolution image by fusing the low-resolution images, based on reasonable assumptions or prior knowledge about the observation model that maps the high-resolution image to the low-resolution ones. The fundamental reconstruction constraint for SR is that the recovered image, after applying the same

generation model, should reproduce the observed low resolution images. However, SR image reconstruction is generally a severely ill-posed problem because of the insufficient number of low resolution images, ill-conditioned registration and unknown blurring operators, and the solution from the reconstruction constraint is not unique. While simple interpolation methods such as Bilinear or Bicubic interpolation tend to generate overly smooth images with ringing and jagged artifacts, interpolation by exploiting the natural image priors will generally produce more favorable results. However, they are limited in modeling the visual complexity of the real images.

For natural images with fine textures or smooth shading, these approaches tend to produce watercolor-like artifacts. A third category of SR approach is based on machine learning techniques, which attempt to capture the co-occurrence prior between low-resolution and high-resolution image patches.

This approach is motivated by recent results in sparse signal representation, which suggest that the linear relationships among high-resolution signals can be accurately recovered from their low-dimensional projections. Although the super-resolution problem is very ill-posed, making precise recovery impossible, the image patch sparse representation demonstrates both effectiveness and robustness in regularizing the inverse problem. To be more precise, let  $D \in \mathbb{R}^{n \times K}$  be an over complete dictionary of  $K$  atoms ( $K > n$ ), and suppose a signal  $x \in \mathbb{R}^n$  can be represented as a sparse linear combination with respect to  $D$ .

## II. RELATED WORKS

### Quaternions

Quaternionic space, denoted as  $H$ , is an extension of the complex space  $C$  using three imaginary parts. A quaternion  $q$  in  $H$  is defined as:  $q = q_a + q_b i + q_c j + q_d k$ , with  $q_a, q_b, q_c, q_d \in \mathbb{R}$  and with the imaginary units defined as:  $i^2 = j^2 = k^2 = -1$ ,  $ij = k$ ,  $ji = -k$ ,  $ik = j$ ,  $ki = -j$ , and  $jk = i$ ,  $kj = -i$ . The quaternionic space is characterized by its noncommutativity:  $q_1 q_2 \neq q_2 q_1$ . The scalar part is  $S(q) = q_a$ , and the vectorial part is  $V(q) = q_b i + q_c j + q_d k$ . If its scalar part is null, a quaternion is said to be pure and full otherwise. The conjugate  $q^*$  is defined as:  $q^* = S(q) - V(q)$  and we have  $(q_1 q_2)^* = q_2^* q_1^*$ .

### The shift-invariant case

The shift-invariant case, we want to sparsely code the signal  $y$  as a sum of a few short structures, named kernels, that are characterized independently of their positions. This is usually applied to time series data, and this model avoids the block effects in the analysis of largely periodic signals, and provides a compact kernel dictionary.

The spikegram for quaternionic decompositions real coding coefficients  $x_l, t$  are displayed by a time-kernel representation called a spikegram. This condenses three indications:

- The temporal position  $t$  (abscissa),
- The kernel index  $l$  (ordinate),
- The coefficient amplitude  $x_l, t$  (gray level of the spike).

This presentation allows an intuitive readability of the decomposition. With complex coefficients, the coefficient modulus is used for the amplitude, and its argument gives the angle, which is written next to the spike.

## III. PROPOSED METHODOLOGY

The proposed model represents the color image as a quaternion matrix, where a quaternion-based dictionary learning algorithm is presented using the K-quaternion singular value decomposition (QSVD) (generalized K-means clustering for QSVD) method. We will show that with the help of QSVD, we can obtain a structured sparse representation model and an effective dictionary learning algorithm for color images. Algorithm for quaternion extension in consideration of its high efficiency, to design the QOMP (quaternion orthogonal matching pursuit) algorithm. Because it has the same framework with OMP, QOMP is still a greedy algorithm — the more nonzero coefficients we obtain, the smaller reconstruction residual we have.

## IV. PROPOSED METHODOLOGY PROCEDURE

### Quaternionic Orthogonal Matching Pursuit

It presents the Q-OMP, the quaternionic extension of the OMP. There are different implementations of the OMP projection. In the following, the block matrix inversion method is extended. First the quaternionic space and notations are outlined, and then detail the Q-OMP algorithm.

**Algorithm 1 :**  $x = \text{OMP}(y, \Phi)$

```

1: initialization :  $k = 1$ ,  $\varepsilon^0 = y$ , dictionary  $D^0 = \emptyset$ 
2: repeat
3:   for  $m \leftarrow 1, M$  do
4:     Inner Products :  $C_m^k \leftarrow \langle \varepsilon^{k-1}, \phi_m \rangle$ 
5:   end for
6:   Selection :  $m^k \leftarrow \arg \max_m |C_m^k|$ 
7:   Active Dictionary :  $D^k \leftarrow D^{k-1} \cup \phi_{m^k}$ 
8:   Active Coefficients :  $x^k \leftarrow \arg \min_x \|y - D^k x\|_2^2$ 
9:   Residue :  $\varepsilon^k \leftarrow y - D^k x^k$ 
10:   $k \leftarrow k + 1$ 
11: until stopping criterion

```

### QUATERNIONI MATRIX

Quaternion matrix is a matrix whose elements are quaternion's.

### Matrix operations

The quaternion's form a non commutative ring, and therefore addition and multiplication can be defined for quaternion matrices as for matrices over any ring.

### Addition.

The sum of two quaternion matrices  $A$  and  $B$  is defined in the usual way by element-wise addition:

$$(A + B)_{ij} = A_{ij} + B_{ij}.$$

### Multiplication.

The product of two quaternion matrices  $A$  and  $B$  also follows the usual definition for matrix multiplication. For it to be defined, the number of columns of  $A$  must equal the number of rows of  $B$ . Then the entry in the  $i$ th row and  $j$ th column of the product is the dot product of the  $i$ th row of the first matrix with the  $j$ th column of the second matrix. Specifically:

$$(AB)_{ij} = \sum_s A_{is} B_{sj}.$$

For example, for

$$U = \begin{pmatrix} u_{11} & u_{12} \\ u_{21} & u_{22} \end{pmatrix}, \quad V = \begin{pmatrix} v_{11} & v_{12} \\ v_{21} & v_{22} \end{pmatrix},$$

the product is

$$UV = \begin{pmatrix} u_{11}v_{11} + u_{12}v_{21} & u_{11}v_{12} + u_{12}v_{22} \\ u_{21}v_{11} + u_{22}v_{21} & u_{21}v_{12} + u_{22}v_{22} \end{pmatrix}.$$

Since quaternion multiplication is non commutative, care must be taken to preserve the order of the factors when computing the product of matrices. The identity for this multiplication is, as expected, the diagonal matrix  $I = \text{diag}(1, 1, \dots, 1)$ . Multiplication follows the usual laws of associativity and distributivity.

### Determinants

There is no natural way to define a determinant for (square) quaternion matrices so that the values of the determinant are quaternion.[2] Complex valued determinants can be defined however.[3] The quaternion  $a + bi + cj + dk$  can be represented as the  $2 \times 2$  complex matrix

$$\begin{bmatrix} a + bi & c + di \\ -c + di & a - bi \end{bmatrix}$$

This defines a map  $\Psi_{mn}$  from the  $m$  by  $n$  quaternion matrices to the  $2m$  by  $2n$  complex matrices by replacing each entry in the quaternion matrix by its  $2$  by  $2$  complex representation. The

complex valued determinant of a square quaternion matrix  $A$  is then defined as  $\det(\Psi(A))$ . Many of the usual laws for determinants hold; in particular, an  $n$  by  $n$  matrix is invertible if and only if its determinant is nonzero.

### Color Image Reconstruction

The proposed sparse model is compared with the other model for color image reconstruction. The dataset for training consists of 50,000 image sample patches of size  $8 \times 8$ , which are randomly selected from a wide variety of animal images with different scenes. Then the dictionaries using K-SVD and K-QSVD separately on the same training samples is trained. In order to keep a reasonable computational complexity, both dictionaries are relatively small with 256 atoms. To provide comparison of our K-QSVD sparse model and Elad's K-SVD sparse model we randomly pick 20 images and concatenate them as a full image for reconstruction. We first compute the PSNR(dB) values over different sparse parameter  $L$  for both models.

The quaternion-based sparse model is able to present higher PSNR values than the model in (24) with the same sparse parameter. The advantage becomes even greater with the increasing number of atoms used. The number of atoms to be used under the same PSNR is also compared. An interesting phenomenon is observed that the advantage of K-QSVD becomes even more obvious when more atoms are allowed to be used. This is due to the lower intra-redundancy between the channel components of each atom and the lower inter-redundancy between each pair of atoms in the quaternion-based dictionary.

### Color Image Denoising

Another common application of sparse representation is denoising. The denoising problem can be formulated as the minimization of the following objective function:

$$\{\hat{\mathbf{D}}, \hat{\mathbf{a}}_{ij}, \hat{\mathbf{X}}\} = \min_{\mathbf{D}, \mathbf{a}_{ij}, \mathbf{X}} \{ \lambda \|\hat{\mathbf{X}} - \mathbf{Y}\|_2^2 + \sum_{i,j} \mu_{ij} \|\hat{\mathbf{a}}_{ij}\|_0 + \sum_{i,j} \|\hat{\mathbf{D}} \hat{\mathbf{a}}_{ij} - R_{ij} \hat{\mathbf{X}}\|_2^2 \},$$

where  $\hat{\mathbf{X}}$  is the estimation of  $\mathbf{X}$ , and the dictionary  $\hat{\mathbf{D}}$  of size  $n \times K$  is the estimation of the optimal dictionary which leads to the sparsest representation of the recovered image patches. The indices  $[i, j]$  mark the location of patches, thus  $R_{i,j}$  is the operator extracting the  $\sqrt{n} \times \sqrt{n}$  square patch at coordinates  $[i, j]$  from  $\mathbf{X}$ , and the vector  $\mathbf{a}_{i,j}$  of size  $K \times 1$  is the coefficient vectors for the patch at index  $[i, j]$ .

The first term enforces the likelihood that demands proximity between  $\mathbf{X}'$  and  $\mathbf{Y}'$ . The second and the third terms impose the image prior, assuming each quaternion patch can be sparsely represented without noise over dictionary  $\mathbf{D}'$ .

The solution is an extension with all algebra operations in quaternion system, where the key part for suppressing noise falls on the QOMP implementation,

$$\min_{\hat{\mathbf{a}}_{ij}} \|\hat{\mathbf{a}}_{ij}\|_0, \quad \text{s.t. } \|\hat{\mathbf{D}}_{ij} \hat{\mathbf{X}} - R_{ij} \hat{\mathbf{X}}\|_2^2 \leq n(C\sigma)^2,$$

which stops searching the best candidate atom once the approximation reaches the sphere of radius.

### Color Image Inpainting

Image inpainting refers to filling the missing information in an image. Limited by the patch size, the learning-based method can only handle small holes. In this paper, we focus on filling missing areas within the order of 30 pixels. We randomly choose one full image which is damaged by randomly deleting a fraction  $r$  of the pixels, usually  $r$ . Our goal is to re-fill them. The workflow of the proposed color image inpainting:

We only consider the projections of non-corrupted pixels onto dictionary in the QOMP algorithm. The coefficient vector for each patch  $\mathbf{p}$  can be estimated only on the non-corrupted pixels  $\mathbf{x}_p$  using the pruned dictionary  $\mathbf{D}'_p$  by selecting corresponding rows of  $\mathbf{D}$ . The computed coefficient vector  $\mathbf{a}_p$  can be shared with those missing pixels, considering its validity for the whole complete patch block  $\mathbf{p}$ . It should be noted that another vector sparse representation model is proposed for color image inpainting as well. However, that model requires a channel (gray or color) to be available in advance for estimating the missing channels. In other words, what it does is colorization rather than inpainting.

### Single Color Image Super Resolution

Single image super-resolution refers to the process of obtaining higher-resolution (HR) images  $\mathbf{X}'_H$  from one lower resolution (LR) image  $\mathbf{X}'_L$ . Current image super-resolution methods can be divided into three categories: interpolation based methods, reconstruction-based methods and example based methods.

Among interpolation-based algorithms, bi-linear and bi-cubic are most commonly used but tend to produce blurry and jaggy artifacts. Reconstruction-based methods require the consistency of up-sampled image with the input LR image, where the HR-to-LR degradation process is reversed by various kinds of edge prior models. More recent researches have focused on the third type, i.e., example-based methods, which reconstruct the high frequency band

of LR image using the provided example database. The works exploited the raw patch information from database, whereas our approach finds the sparse representation of the example database, similar to the approach.

### V. CONCLUSION

This project propose a novel sparse model for color image using quaternion matrix analysis. It formulates a color pixel as a vector unit instead of a scalar quantity and consequently overcomes the lack of accuracy describing inter-relationship among color channels. The experiments of reconstruction, denoising, inpainting, and super-resolution on natural color images prove its advantages in effectively accounting for both luminance and chrominance geometry in images. Currently, the usage of the real part of quaternion seems insufficient: for three-channel color space, the real part is simply set to be zero. We believe that the physically meaningful real part will further help us capture color information. In the future, we will further explore the potential extension of quaternion sparse model to four channel color space.

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