

# Performance Analysis Of Multiple Access Relay Channel With Analog Network Coding

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#### **ABSTRACT**

The present day wireless network needs co-operation to enhance the performance at reduced cost. The need for multiple access relaying is also proportionately increasing. The use of analog network coding (ANC) on the multiple access relay channel (MARC) with multiple relay has been under investigation to enhance the performance in a co-operative wireless environment. The performance of the multiple source with multiple relay employing ANC is proposed. The combining techniques like Maximum-Likelihood (ML) and Zero-Forcing (ZF) detector has to be modeled for the above system. A mathematical modeling along with performance increases on error rate is to be estimated.

Keywords— Analog network coding, Maximum-likelihood, Zero-forcing detector, Multiple Access Relay Channel.

#### I. INTRODUCTION

Analog Network Coding (NC) was introduced as a technique for achieving the maximum information flow in networks with multiple nodes, by enabling intermediary nodes to perform coding operations on the incoming packets, such as the exclusive-OR operation [1]. Although initially intended for wired networks, the adoption of NC was subsequently extensive to wireless networks that consist of intermediary nodes between the sources and the destination, such as relays [2]. Among the several NC techniques projected in the literature[3]-[7], various researchers have shown that Analog Network Coding provides a good exchange in terms of computational complication and possible performance [8], [9]. In ANC, in fact, the relays employ them broadcast nature of the wireless channel by simply amplifying and forwarding a superposition of the received signals to them. It requires less computational complexity than its digital counterparts at the relay such as decode-and-forward and compute-and-forward. The price to be remunerated and the performance can be degraded by noise amplification factor at low-SNR originating from using anon-regenerative transmission protocol. To overcome this, ANC is measured to be a good runner transmission technology at high-SNR.

A wireless standards where the operation of relay is distributed [10] is the so-called Multiple Access Relay Channel (MARC), the multiple sources communicate to a common destination with one or multiple relays [6] and used for uplink transmission in cellular networks. In particular, the authors of [11] proved that the ANC achieves the finest diversity multiplexing substitution at high SNR multiplexing gains, as well a s that it outperforms the decode-and-forward and compress-and-forward relaying at low and high SNR multiplexing gain, respectively. In [12], a family of optimal DSTC [15] for application to the two-user MARC is proposed, which achieves the best possible

diversity-multiplexing tradeoff. In [13], the authors distinguish the most favorable relay amplification factor and the resulting optimal rate regions in multi-hop MARCs.

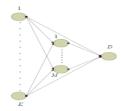


Fig.1The Multiple Access Relay Channel

From the above, ANC is measured to be a interested relaying protocol for high-SNR transmission. In this operating regime, the communication-theoretic performance of cooperative relaying protocols are frequently quantify in terms of coding gain and diversity order [16]. In [17], a Detector as Maximum-Likelihood (ML) at the destination is considered and the diversity order of ANC with best relay selection and DSTC coding transmission is studied. In this paper, we derived the Symbol Error Rate(SER) with low complexity diversity order and coherent demodulation at the destination. Since in some applications opinion channel may not be available, in the present paper we focus our thought on a repetition-based relaying protocol. The present paper is an complete version of where linear receivers are not considered. Hence the SER with power allocation provide the high and mighty repetition coding and ML detection. Hence it is applicable to bidirectional communication protocols using two sources and variable-gain relays.

#### II. SYSTEM MODEL

Consider the MARC in Fig. 1, which may find application for uplink transmission in next generation cellular networks. A co-operative relay networks consists of K sources communicate with a single destination and M fixed-gain and half-duplex relays through direct transmissions. And hence assume that all the nodes are equipped with a single antenna and that the channel is narrow and block-fading. Let  $\sigma_{kD}^2$ ,  $\sigma_{km}^2$  and  $\sigma_{mD}^2$  where k = 1,2,3...K and m = 1,2,...M denote the path-loss coefficients of these links are source to relay, relay to destination and source to destination. They depend on the transmission distance between the nodes depend on direct and indirect links. Then,  $\tilde{h}_{kD} \sim CN\left(0,\sigma_{kD}^2\right)$  denotes the channel coefficient from the  $k_{th}$  source to the destination,  $\tilde{h}_{km} \sim CN\left(0,\sigma_{km}^2\right)$  denotes the channel coefficient from the  $k_{th}$  source to the channel coefficient from and PR, respectively. The communication protocol consists of two phases:

#### A. 1st Phase – Direct Transmission from the sources to the relays:

In the first phase, which has duration of 1 time slot, the K sources simultaneously broadcast their modulated packets to the destination and the relays. Hence, the received signal at the destination, which we denote as  $y_{SD}$ , is given by

$$y_{Dm} = \sqrt{P_R \sigma_{RD}^2} r_m y_m \tilde{f}_m + n_{Dm} \tag{1}$$

where  $\tilde{h}_{kD} \sim CN(0,1)$  and  $n_{SD} \sim CN(0,1)$  denotes the Additive White Gaussian Noise (AWGN) during the 1st time slot at the destination. The received signal  $y_m$  at the  $m_{th}$  relay is given by

$$y_m = \sum_{k=1}^K \sqrt{P_S \sigma_{SR}^2} \tilde{h}_{km} s_k + n_m \tag{2}$$

where  $\tilde{h}_{km} \sim CN(0,1)$  and  $n_m \sim CN(0,1)$  is the AWGN realization at the  $m_{th}$  relay. This signal model assumes that there is perfect synchronization at the relays of the incoming signals from the sources. Imperfect synchronization at the relays is an interesting future scope.

#### B. 2nd Phase - Transmission from the relays to the destination:

During the second phase, the relay has fixed-gain amplify their received signal and forward it to the destination. When considering a repetition-based transmission protocol from the relays, it will implies that the relays sequentially forward their signal to the destination in G non-overlapping time slots. From the G+1 time slots are required between source and destination for the end-to-end communication. Although Relaying with repetition protocol is spectrally-inefficient, it constitutes a multiple relay with low-complexity option for the diversity potential of having multiple relays.

The gain  $r_m$  of the relays, which normalizes their received power of sources with average transmission received power, is given by [22].

$$r_m = \sqrt{\frac{1}{KP_S \sigma_{SR}^2 + 1}} \tag{3}$$

Consequently, the received signal at the destination from each of the relays can be formulated as

$$y_{Dm} = \sqrt{P_R \sigma_{RD}^2} r_m y_m \tilde{f}_m + n_{Dm}$$

$$y_{Dm} = \sqrt{\frac{P_S \sigma_{SR}^2 P_R \sigma_{RD}^2}{K P_S \sigma_{SR}^2 + 1}} \tilde{f}_m \sum_{k=1}^K \tilde{h}_{km} S_k + \tilde{n}_{Dm}$$

$$(4)$$

Where  $\tilde{f}_m \sim CN(0,1)$  and  $n_{Dm} \sim CN(0,1)$  is the AWGN at each of the G time slots of the relay transmission phase at the destination  $\tilde{n}_{Dm} \sim CN\left(0, \frac{P_R\sigma_{RD}^2}{KP_S\sigma_{SR}^2+1} \left|\tilde{f}_m\right|^2+1\right)$ .

#### III. PERFORMANCE ANALYSIS BY CONSIDERING THE ABSENCE OF DIRECT LINKS

In this analysis, we first derive approximate high-SNR closed-form formulas for the SER of ML and ZF receivers by assuming that  $\sigma_{SD}^2 = 0$  i.e., no direct links are available. The mathematical expressions which was projected at each receiver that expose the coding gain and diversity order. Subsequently, and the performance behavior of the coding gain ratio of ML and ZF receivers with respect to the number of sources, the number of relays, and the modulation order.

Let  $\sigma_{SD}^2 = 0$ , then, the channel matrix is reduced to the matrix product FH, where  $F \in C^{M \times M}$  is a diagonal matrix with  $\tilde{f}_1, ..., \tilde{f}_M$  being the elements of its main diagonal and Consequently, the received signal vector, which we denote as y, is given by

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$$y = \sqrt{\frac{P_S \sigma_{SR}^2 P_R \sigma_{RD}^2}{K P_S \sigma_{SR}^2 + 1}} F H_S + \tilde{n}_D$$
(5)

Where, 
$$\tilde{n}_D = (\tilde{n}_{D1} ... \tilde{n}_{DM})^T$$

#### A. Ml Detector

To communicate over the AWGN channel, we first collecting the task of detecting a set of M transmitted symbols with a set of N pilot signals. Our knowledge was corrupted by the non-ideal communication channel, typically modeled as a linear MIMO system followed by an additive white Gaussian vector.

A linear MIMO communication system diagram shows that the following discrete time signals: transmitted symbol vector  $S \in \chi^M$ , channel matrix  $H \in R^{M \times N}$ , additive noise vector  $n \in \Re^N$ , received vector  $v \in \Re^N$ , and detected symbol vector  $\hat{s} \in \Re^N$ .

$$s_* \cong \arg\max_{S \in \chi^M} \left( s \text{ was sent } | v \text{ is observed} \right)$$

To assist us in achieving our goal, we draw the transmitted symbols from a known alphabet  $\chi = \{x_1, ..., x_B\}$  of size B. The detector's role is then to choose one of the B M possible transmitted available data as symbol vectors. Our perception correctly suggests that an optimal detector should return  $\hat{s} = s_*$ , a posteriori probability of the symbol vector which is sent given the observed signal vector v.

$$s_* \cong \arg\max_{S \in \mathcal{X}^M} \left( vis \ observed \mid \left( swas \ sent \right) \right)$$

The equation is observed as rule of Maximum A-posteriori Probability (MAP) detection is (s was sent) is constant. A detector that satisfy the above optimal condition is called a Maximum Likelihood detector(ML). And hence assume that the additive noise Gaussian noise and it is expressed as the ML detection problem as the minimization the value of square with Euclidean distance metric to a objective vector v over an M -dimensional discrete search set:

$$s_* = \arg\min_{s \in \gamma M} \left| v - Hs \right|^2$$

Where, borrowing expressions from the optimization prose we call the elements of s optimization variables and  $\left|v - Hs\right|^2$  the objective function. By employing the ML-optimal detection criterion, the destination can jointly detect the transmitted symbols from the sources as

$$s_{\text{det}} = \underset{\hat{s}_{k}}{\operatorname{arg \, min}} \sum_{m=1}^{M} \frac{\left| y_{D_{m}} - \sqrt{\frac{P_{S}\sigma^{2} SR P_{R}\sigma^{2} RD}{KP_{s}\sigma^{2} SR + 1}} \tilde{f}_{m} \sum_{k=1}^{K} \tilde{h}_{km} \hat{s}_{k} \right|^{2}}{\frac{P_{S}\sigma^{2} RD}{KP_{s}\sigma^{2} SR + 1} \left| \tilde{f}_{m} \right|^{2} + 1}$$

(6)

Where  $s_{\text{det}} = \left(s_{1_{\text{det}}}, \dots, s_{K_{\text{det}}}\right)^{\text{T}}$  is the detected symbol vector. To derive an analytical closed-form framework for the union bound of the SER per source. Let  $\left\{s_q\right\}$  is the set of all Q possible symbols transmitted from a various source, and consider to be equal likely. Furthermore, let  $\left\{s\right\}$  denote the set of the  $Q^K$  symbol vectors to be transmitted from the K sources, where  $\left\{s_i\right\}$  defines a subset of  $\left\{s\right\}$  in which the symbol vectors have  $s_q$  transmitted from the  $k_{th}$ 

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source. Thus in total there are  $Q^{K-1}$  such vectors. Also, let  $\{s_j\}$  denote the set of  $Q^K - Q^{K-1}$  symbol vectors in which the symbol transmitted from the  $k_{th}$  source is different from  $s_q$ . Assuming that all the sources has equal modulation order Q, the SER for ML detection in the union bound, which we denote as  $SER_{ML}$  is given by

$$SER_{ML} \leq Q^{-K} \underbrace{\sum_{q=1}^{Q} \sum_{i=1}^{Q^{K}-1} Q^{K} - Q^{K-1}}_{q=1} PEP_{s_q,ij}$$

Where,  $PEP_{s_q,ij}$  denotes the symbol vector of pair-wise error probability (PEP) for detecting the symbol vector  $s_j$  when the vector  $s_i$  is transmitted, where Q denotes the modulation order and (i, j) denotes the random vector symbol which is to be considered while using in Maximum Likelihood (ML) detector.

#### B. Zero-Forcing(ZF) detector

Zero Forcing detectors is a linear coherent detector used in communication systems which proceeds the inverse of the frequency response of the particular channel. Due to the high complexity of the non-linear ML detector Considering that the MARC with ANC is equivalent to a spatial multiplexing MIMO channel and a simple detector as same as that belongs to this category of detector is the ZF detector. In this case, the constraint  $K \leq M$  needs to hold due to matrix inversion properties. These macro-cell users that are included to be as analog network-coded in the uplink transmission. For this ZF receiver, considering that the channel matrix vector is FH, the ZF-equalized received signal vector  $y_{eq}^{ZF}$  is given by

$$\begin{aligned} y_{eq}^{ZF} &= G_{ZF} \, y \\ &= \sqrt{\frac{P_S \sigma_{SR}^2 P_R \sigma_{RD}^2}{K P_S \sigma_{SR}^2 + 1}} s + \hat{n}_{ZF} \end{aligned}$$

where.

$$G_{ZF} = \left[ \left( FH \right)^{H} FH \right]^{-1} \left( FH \right)^{H} \tag{7}$$

### IV. PERFORMANCE ANALYSIS ASSUMING THE PRESENCE OF DIRECT LINKS

By assuming the presence of direct links at the destination  $\sigma_{SD}^2 \neq 0$ . More specifically, they

- *i*) Provide a closed form expression of the union bound of the SER per source, which are obtained from the coding gain and diversity order.
- $\ddot{u}$ ) To obtain the SER of closed-form expression is difficult proceeds in the case of ML and ZF receivers. Before proceeding with the analysis, we note that there are two differences with respect to the system model without direct

links 
$$\sigma_{SD}^2 = 0$$
. The first element  $\sqrt{P_S \sigma_{SD}^2} \sqrt{\frac{KP_S \sigma_{SR}^2 + 1}{P_S \sigma_{SR}^2 P_R \sigma_{RD}^2}}$  of  $F_{D.Links}$ 

#### A. ML Detector

By demonstrating the ML detector, the detected symbol vector  $s_{\text{det}} = (s_{1_{\text{det}}}, ..., s_{K_{\text{det}}})^T$  is given by

$$s_{\text{det}} = \arg\min_{\hat{s}_{k}} \left| y_{SD} - \sqrt{P_{S} \sigma_{SD}^{2} \sum_{k}^{K} \hat{h}_{kD} \hat{s}_{k}} \right|^{2}$$

$$+ \sum_{m=1}^{M} \frac{\left| y_{D_{m}} - \sqrt{\frac{P_{S} \sigma^{2} S_{R} P_{R} \sigma^{2} R_{D}}{K P_{s} \sigma^{2} S_{R} + 1}} \tilde{f}_{m} \sum_{k=1}^{K} \tilde{h}_{km} \hat{s}_{k} \right|^{2}}{\frac{P_{S} \sigma^{2} R_{D}}{K P_{s} \sigma^{2} S_{R} + 1} \left| \tilde{f}_{m} \right|^{2} + 1}$$
(8)

From the (15) ,This is an approximate formula for the ML detector for Pair-wise Error probability(PEP) , which we denote as  $PEP_{s_{q,ij}}^{Direct}$  Links and it is given by

$$PEP_{s_{q,ij}}^{Direct} \quad Links = \frac{1}{12} \left[ \frac{1}{b+1} + \frac{bc}{a(b+1)^2} Z \left( \frac{c}{a(b+1)} \right) \right]^M \times \frac{4}{4 + P_S \sigma_{SD}^2 \left\| \Delta s_{i,j} \right\|^2} + \frac{1}{6} \left[ \frac{3}{4b+3} + \frac{12bc}{a(4b+3)^2} Z \left( \frac{3c}{a(4b+3)} \right) \right]^M \times \frac{3}{3 + P_S \sigma_{SD}^2 \left\| \Delta s_{i,j} \right\|^2}$$

and hence the union bound of the SER per source of the ML detector with respect to the direct links are obtained.

#### **ZF** Detector

The performance analysis of the SER of ZF detectors involves the computation

$$E\left\{\widetilde{n}_{D.Links}\widetilde{n}^{H}D_{D.Links}\right\} = diag\left(\frac{P_{R}\sigma_{RD}^{2}}{KP_{S}\sigma_{SR}^{2}+1}....\frac{P_{R}\sigma_{RD}^{2}}{KP_{S}\sigma_{SR}^{2}+1}\right)$$

the same elements does not have the main diagonal (due to the last element that corresponds to the direct links) and hence for the ZF detectors, cannot be obtained. Consequently,  $E \left\{ \tilde{p}_{DD.Links} \tilde{n}^H D_{D.Links} \right\}$  does not uses the same elements as a main diagonal due to the last element of direct links. The zero-forcing equalizer removes all ISI, and is ideal when the channel is noiseless. However, when the channel is noisy, the zero-forcing equalizer will amplify the noise greatly at frequencies f where the channel response  $H(j2\pi f)$  has a small magnitude (i.e. near zeroes of the channel) in the attempt to invert the channel completely. A more balanced linear equalizer in this case is the minimum mean-square error equalizer, which does not usually eliminate ISI completely but instead minimizes the total power of the noise and ISI components in the output. Hence, it is concluded that the ZF detector is mainly depends on the number of sources and not on the destination with equal power allocation with respect to the source, relay and destination. And for the performance of ZF detector is not to be detectable due to the insufficient elements in the diagonal value of symbol vector

#### V. NUMERICAL RESULTS

#### A. ML Detector

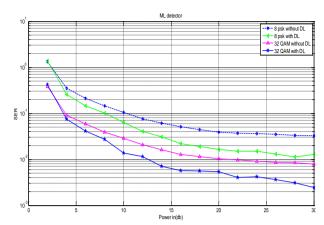


Fig.3 SER Performance for ML Detector

From the above figure 3, it describes the SER performance of Maximum likelihood(ML) Detector for PSK and QAM modulation by using direct and indirect links. The SER of the ML detector depends only on  $p_R$  and  $\sigma_{SR}^2$ , which means that the higher these values and lower the symbol error rate(SER).

#### B. ZF Detector

From the below figure 4, it describes the SER performance of Zero-Forcing (ZF) Detector for 4-psk and 8-psk by using without direct links. The diversity order of the ZF detector depends on the number of sources and decreases as the number of sources increases. From the above theory, the performance can be obtained by assuming the presence of direct links is mainly depends on the number of sources and relaywhich is to be used

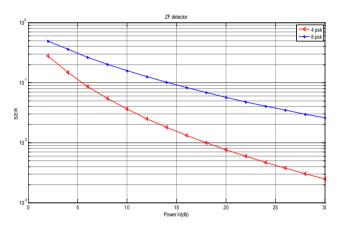
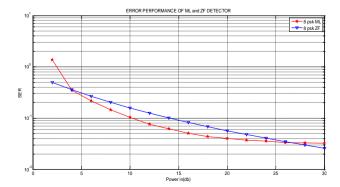


Fig.4 SER Performance of ZF Detector

C.Comparsion of Maximum likelihood (ML) and Zero- Forcing(ZF) detection



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#### Fig.5 SER performance of ML and ZF Detector

From the above figure 5, it describes the SER performance of Maximum likelihood(ML) and Zero-Forcing Detector for PSK modulation. Hence the performance gap increases as the number of sources increases between the ML and ZF detectors since the diversity order of the former detector does not depend on the number of sources. In this the channel quality of the source to relay links is better than the channel quality of relay to destination links. From this analysis, it concludes that the SER with equal power allocation for ML and ZF detector has been simulated.

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